10.8 Composite Solids

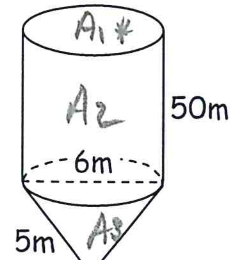
Worksheet

Name________________________

Period_____

Keep answers in terms of π, unless indicated to round to the nearest tenth.

A well, with a cylindrical wall of 50 m. and a diameter of 6 m., is dug. The bottom of the well is tapered to a cone with slant height of 5 m. Building codes require the well to be covered.*



1. Find the height of the cone.

3m, 4m, 5m Pythagorean triplet

$B = \sqrt{c^2 - A^2}$
$= \sqrt{25m^2 - 9m^2}$
$= \sqrt{16m^2}$
$= 4m$

2. Find the amount of material needed to construct the surface area of the well.

$AT = A_1 + A_2 + A_3$
$= 28.26m^2 + 942m^2 + 47.1m^2$
$= 1017.36m^2$
$\therefore$ Total surface area is $1017.36m^2$

$A_1 = \pi r^2$
$= 3.14 \times 3m \times 3m$
$= 3.14 \times 9m^2$
$= 28.3m^2$

$A_2 = 2\pi r \times H$
$= 2 \times 3.14 \times 3m \times 50m$
$= 942m^2$

$A_3 = \pi r s$
$= 3.14 \times 3m \times 5m$
$= 47.1m^2$

3. Find the volume of water that this well could hold.

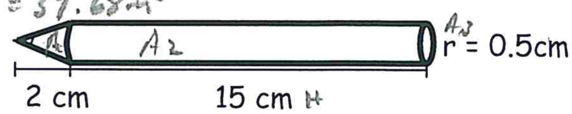
$VT = VA_2 + VA_3$
$= 1413m^3 + 37.68m^3$
$= 1450.68m^3$

$\therefore$ The total volume is $1450.7m^3$

$VA_2 = A_1 \times H$
$= 28.26m^2 \times 50m$
$= 1413m^3$

$VA_3 = \frac{A_1 \times 4m}{3}$
$= \frac{28.26 \times 4m^2}{3}$
$= \frac{113.04m^3}{3}$
$= 37.68m^3$

A small lead pencil has a cylindrical base and a conical point.



4. Find the slant height of the cone. (Use the Pythagorean theorem.)
Round to the nearest tenth.

2cm, .5cm

$S = \sqrt{a^2 + b^2}$
$= \sqrt{(0.5m)^2 + (2cm)^2}$
$= \sqrt{0.25cm^2 + 4cm^2}$
$= 2.06cm$

The slant height to the nearest tenth 2.1cm

5. Find the surface area of the whole pencil.

$SAT = A_1 + A_2 + A_3$
$= 3.297cm^2 + 47.1cm^2 + 0.785cm^2$
$= 51.182cm^2$

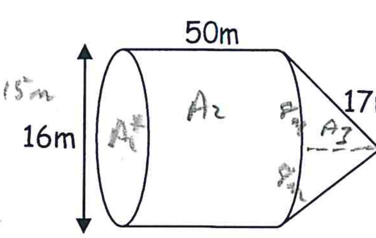
$A_1 = \pi r s$
$= 3.14 \times .5cm \times 2.1cm$
$= 3.297cm^3$

$A_2 = 2\pi r H$
$= 3.14cm \times 15cm$
$= 47.1cm^2$

$A_3 = \pi r^2$
$= 3.14 \times .5 \times .5$
$= 0.785cm^2$

$\therefore$ The surface area of the pencil is $51.2cm^2$ (to the nearest 10th)

A rocket has dimensions as shown to the right. The entire rocket is filled with fuel.



6. Find the height of the cone?

17m, 15m, 8m

$H = \sqrt{(17m)^2 - (8m)^2}$
$= \sqrt{289m^2 - 64m^2}$
$= \sqrt{225m^2}$
$H = 15m$

The height is 15m

7. Find the amount of material needed to construct the surface area of the rocket.

$AT_{no\ end} = A_2 + A_3$
$= 2512m^2 + 427.04m^2$
$= 2939.04m^2$

$AT_{end} = A_1 + A_2 + A_3$
$= 200.96m^2 + 2939.04m^2$
$= 3140m^2$

$A_1 = \pi r^2$
$= 3.14 \times (8m)^2$
$= 200.96m^2$

$A_2 = \pi D H$
$= 2512m^2$

$A_3 = \pi r s$
$= 3.14 \times 8m \times 17m$
$= 427.0m^2$

$\therefore$ Total surface/amount of material is $2939.04m^2$

8. What is the volume of fuel that the rocket may contain?

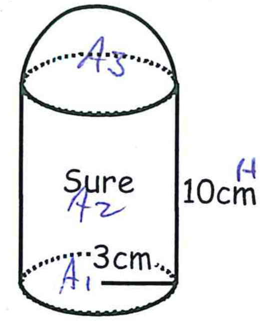
$VT = VA_2 + VA_3$
$= 10048m^3 + 1004.8m^3$
$= 11052.8m^3$

Total volume is $11052.8m^3$

$VA_2 = A_1 \times H$
$= 200.96m^2 \times 50m$
$= 10048m^3$

$VA_3 = \frac{A_1 \times H}{3}$
$= \frac{3014.4m^3}{3}$
$= 1004.8m^3$

9. A stick of Sure Solid deodorant is shown to the right.
Find the amount of material (surface area) needed to construct the container.



$A_T = A_1 + A_2 + A_3$
$= 28.26cm^2 + 188.4cm^2 + 56.52cm^2$
$= 273.18cm^2$
$\cong 273.2cm^2$

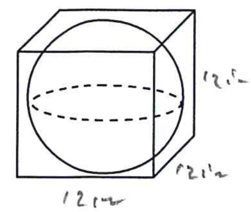
$A_1 = \pi r^2$
$= 3.14 \times 3cm \times 3cm$
$= 28.26cm^2$

$A_2 = 2\pi r H$
$= 2 \times 3.14 \times 3cm \times 10cm$
$= 188.4cm^2$

$A_3 = \frac{1}{2}$ sphere
$= \frac{1}{2}\, 4\pi r^2$
$= 2\pi r^2$
$= 2\, A_1$
$= 2 \times 28.26\ cm^2$
$= 56.52cm^2$

∴ The amount of material needed to construct the container is $273.2cm^2$

The sphere just fits in a cube with edges 12 in. long.



10. What is the radius of the sphere?
The radius is half the diameter, so it is 12in ÷ 2 or 6in.

11. What is the volume of the space between the sphere and cube?
Round to the nearest tenth.

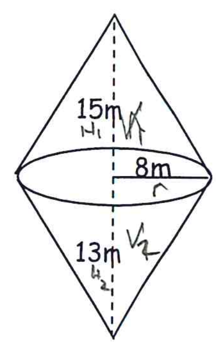
$V_{space} = V_{cube} - V_{sphere}$
$= 1728in^3 - 904.32in^3$
$= 823.68in^3$

$V_{cube} = s^3$
$= (12in)^3$
$= 1728in^3$

$V_{sphere} = \frac{4}{3}\pi r^3$
$= \frac{4}{3} \times 3.14 \times 6in \times 6in \times 6in$
$= 904.32in^3$

∴ The space left is $823.7in^3$

12. Find the volume of the composite solid. **Round to the nearest tenth.**



$A_{face} = \pi r^2$
$= 3.14 \times 8$
$= 200.96$

$V_T = V_1 + V_2$
$= 1004.8m^3 + 870.83m^3$
$= 1875.63m^3$

∴ The volume is $1875.6m^3$.

$V_1 = \frac{A_{face} \times H_1}{3}$
$= \frac{200.96 \times 15m}{3}$
$= \frac{3014.4m^3}{3}$
$= 1004.8m^3$

$V_2 = \frac{A_{face} \times H_2}{3}$
$= \frac{200.96m^2 \times 13m}{3}$
$= \frac{2612.48m^3}{3}$
$\approx 870.83m^3$

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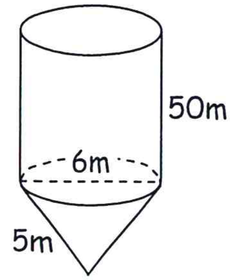
Building codes require the well to be covered.



1. Find the height of the cone.

$b = \sqrt{c^2 - a^2}$

$\boxed{h = 4m}$

2. Find the amount of material needed to construct the surface area of the well.

$S_c = \pi r L = \pi(3m)(5m) = 15\pi\ m^2$

$S_{CYL.} = \pi r^2 + 2\pi r h = 9\pi\ m^2 + 2\pi(3m)(50m) = 309\pi\ m^2$

$S_T = S_c + S_{CYL} = \boxed{324\pi}$

3. Find the volume of water that this well could hold.

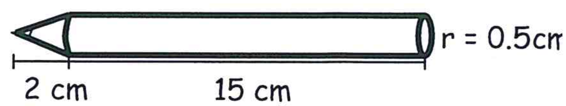
$V_T = V_C + V_{CYL.}$

$V_{CYL.} = A_f \times h = \pi r^2 h = 450\pi\ m^3$

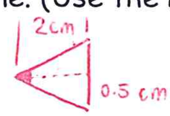
$V_C = \frac{A_f \times h}{3} = \frac{\pi r^2 h}{3} = \frac{\pi(3m)^2(4m)}{3} = 12\pi\ m^3$

$\therefore \boxed{V_T = 462\pi\ m^3}$

A small lead pencil has a cylindrical base and a conical point.



4. Find the slant height of the cone. (Use the Pythagorean theorem.)
Round to the nearest tenth.



$a^2 + b^2 = c^2$

$c^2 = (0.5\ cm)^2 + (2\ cm)^2$ $\quad C = \boxed{2.1\ cm}$

5. Find the surface area of the whole pencil.

$S_T = S_{TIP} + S_{CYL.} + S_{END}$

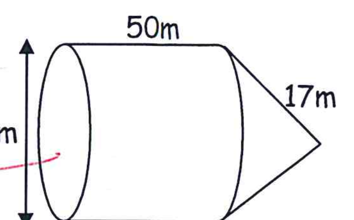
$S_{CYL.} = 2\pi r L = 2\pi(0.5cm)(15\ cm) = 15\pi\ cm^2$

$S_{TIP} = \pi r L = \pi(0.5cm)(2.1cm) = 1.05\pi\ cm^2$

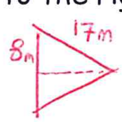
$S_{END} = \pi r^2 = \pi(0.5\ cm)^2 = 0.25\pi\ cm^2$

$\therefore \boxed{S_T = 16.3\pi\ cm^2}$

A rocket has dimensions as shown to the right. The entire rocket is filled with fuel.



6. Find the height of the cone?



$a^2 = c^2 - b^2$

$a = \boxed{h = 15\ m}$

7. Find the amount of material needed to construct the surface area of the rocket.

$S = \pi r^2 = 64\pi\ m^2$

$S_T = S_{CYL} + S_c$

$S_c = \pi r L = \pi(8\ m)(17\ m) = 136\pi\ m^2$

$S_{CYL} = 2\pi r L = 2\pi(8\ m)(50\ m) = 800\pi\ m^2$

NO END CAP

$\therefore \boxed{S_T = 936\pi\ m^2}$

WITH END CAP

$\boxed{S_T = 1000\pi\ m^2}$

8. What is the volume of fuel that the rocket may contain?

$V_T = V_c + V_{CYL}$

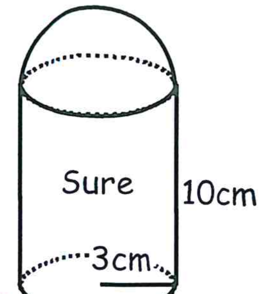
$V_{CYL} = \pi r^2 h = \pi(8m)^2(50\ m) = 3200\pi\ m^3$

or

$V_c = \frac{A_f \times h}{3} = \frac{\pi r^2 h}{3} = \frac{\pi(8m)^2(15m)}{3} = 320\pi\ m^3$

$\therefore \boxed{V_T = 3520\pi\ m^3}$

9. A stick of Sure Solid deodorant is shown to the right.
   Find the amount of material (surface area) needed to construct the container.



$$S_T = \frac{S_{SPHERE}}{2} + S_{CYL} + S_{END}$$

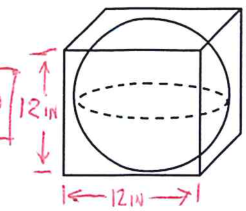
$$\frac{S_{SPHERE}}{2} = \frac{4\pi r^2}{2} = 2\pi r^2 = 2\pi(3\,cm)^2 = 18\pi\ cm^2$$

$$S_{CYL.} = 2\pi r h = 2\pi(3\,cm)(10\,cm) = 60\pi\ cm^2$$

$$S_{END} = \pi r^2 = \pi(3\,cm)^2 = 9\pi\ cm^2$$

$$\therefore \boxed{S_T = 87\pi\ cm^2}$$

The sphere just fits in a cube with edges 12 in. long.



10. What is the radius of the sphere? $d = 12\ in$ $\quad r = \frac{d}{2} = \frac{12\ in}{2} = \boxed{6\,in}$

11. What is the volume of the space between the sphere and cube?
    Round to the nearest tenth.

$$V_{SPACE} = V_{CUBE} - V_{SPHERE}$$

$$V_{SPHERE} = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(6\,in)^3$$

$$= \frac{4}{3}\pi \times 216\ in^3 = 288\pi\ in^3$$

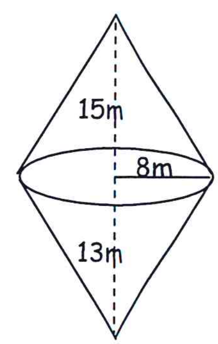
$$V_{CUBE} = L \times W \times H$$

$$= (12\ in)(12\ in)(12\ in)$$

$$= 1728\ in^3$$

$$\therefore V_{SPACE} = 1728\ in^3 - 288\pi\ in^3 \approx \boxed{823.2\,i}$$

12. Find the volume of the composite solid. **Round to the nearest tenth.**



$$V_T = V_{C1} + V_{C2}$$

$$V_{C1} = \frac{\pi r^2 h_1}{3} = \frac{\pi(8\,m)^2(15\,m)}{3} = 320\pi\ m^3$$

$$V_{C2} = \frac{\pi r^2 h_2}{3} = \frac{\pi(8\,m)^2(13\,m)}{3} = \frac{832}{3}\pi\ m^3$$

$$V_T = 320\pi\ m^3 + \frac{832}{3}\pi\ m^3$$

$$= \frac{960\pi}{3}\ m^3 + \frac{832\pi}{3}\ m^3$$

$$= \frac{1792\pi}{3}\ m^3 = \boxed{1876.6\ m^3}$$